Mansoura University
Faculty of Engineering

Final Exam.
Thursday 4/6/2015

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab | Mathematics 4 (Math 102) | Time allowed: 2 hrs.

Solve as possible as you can (Full mark 50 pts.)

1. (*a*) Classify the point $x = -2$ for the differential equation

$$(x-1)^2(x+2)^3\,y'' + (x^3 - x^2)y' + (x+1)y = 0$$

[3 pts.]

(*b*) Find the series solution of the differential equation

$$y'' - 2x^3y' + 2x^2y = 0, \qquad y(0) = 4,\ y'(0) = 7$$

[7 pts.]

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2. (*a*) Sketch the following function on the interval $[-2\pi, 2\pi]$

$$f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}, \qquad f(x+2\pi) = f(x)$$

Find the Fourier series of this function and evaluate $f\left(\frac{9\pi}{2}\right)$.

[5 pts.]

(*b*) Verify Stokes' theorem for the vector field

$$F = x^2z\,\mathbf{i} + 4xy^2\,\mathbf{j} + z^2\,\mathbf{k}$$

on the solid bounded by the paraboloid $4z = 9 - x^2 - y^2$ and the *xy*-plane. **[5 pts.]**

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3. (*a*) Using the graph in the *xy*-plane, classify the following partial differential equation

$$(x^2-1)u_{xx} + 2xyu_{xy} + (y^2-1)u_{yy} - xu_x - \sin x = 0$$

[3 pts.]

(*b*) Solve the following boundary value problem by the use the technique of separation of variables to

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 \le x \le \pi,\ 0 \le y \le \pi$$

$$u(0,y) = u(\pi, y) = 0, \qquad 0 \le y \le \pi$$

$$u(x,0) = 0, \qquad 0 \le x \le \pi$$

$$\frac{\partial u}{\partial x}(x,\pi) = x, \qquad 0 \le x \le \pi$$

[7 pts.]

P.T.O.

4. ($\boldsymbol{a}$) Use the divergence theorem to evaluate $\iint_S F \cdot dS$, where $F = 5x\,\mathbf{i} - 3y\,\mathbf{j} - 10\,\mathbf{k}$ and S is the surface of the elliptic paraboloid $8z = x^2 + 4y^2$, intercepted by $z = 2$. What is the volume of the described solid of integration? **[5 pts.]**

($\boldsymbol{b}$) Evaluate the following integrals

($\boldsymbol{i}$) $I_1 = \int_0^4 \int_{\sqrt{y}}^2 \left(\frac{\tan x}{x}\right)^2 dx\, dy$ **[5 pts.]**

($\boldsymbol{ii}$) $I_2 = \int_C y^5\, dl$, where C is the upper half of the circle $x^2 + y^2 = 4$ **[5 pts.]**

($\boldsymbol{iii}$) $I_3 = \iint_S (x^2 + y^2 + z^2)^{3/2}\, dS$, where S is the hemisphere $z = \sqrt{9 - x^2 - y^2}$ **[5 pts.]**

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Faculty of Engineering

Final Exam.
Thursday 26/5/2016

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab | Mathematics 4 (Math 102) | Time allowed: 2 hrs.

Solve the following questions (Full mark 50 pts.)

1. (a) [5 pts.] Evaluate the double integral $\int_0^4 \int_{\sqrt{y}}^2 9\sqrt{x^3+1}\, dxdy$.

(b) [5 pts.] Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y + 5$.

(c) [10 pts.] Verify Stokes' theorem for the vector field

$$\vec{F}(x,y,z) = 2y\,\mathbf{i} - 5z\,\mathbf{j} - 3x\,\mathbf{k},$$

and S is the solid bounded by the paraboloid $z = 9 - (x^2 + y^2)$, $z \geq 0$.

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2. (a) [5 pts.] Using the graph in the *xy*-plane, classify the following partial differential equation

$$xu_{xx} + 2u_{xy} + xu_{yy} + 4u_y - 6xy = 0$$

(b) [5 pts.] Build a model that describes the temperature distribution in a rod of length π made of homogeneous metal with constant cross section A that is completely insulated along its lateral edges.

(c) [10 pts.] Use the technique of separation of variables to solve the following boundary value problem:

$$u_t = 4u_{xx}, \qquad 0 \leq x \leq \pi, \quad t \geq 0,$$
$$u_x(0,t) = u_x(\pi,t) = 0, \qquad t \geq 0$$
$$u(x,\pi) = 1 - \frac{x}{\pi}, \qquad 0 \leq x \leq \pi$$

3. (a) [5 pts.] Find Fourier integrals of the function

$$f(x) = \begin{cases} \sin x, & -\pi \leq x \leq \pi \\ 0, & otherwise \end{cases}$$

(b) [5 pts.] Use the divergence theorem to evaluate $\iint_S \vec{F} \cdot dS$, where

$$\vec{F}(x, y, z) = (5x - y\cos y)\,\mathbf{i} + (2y + 4\sin z)\mathbf{j} + (3z + 9e^x)\mathbf{k}$$

and S is the surface of the region R bounded by the solid that lies above the cone $z = \sqrt{3(x^2 + y^2)}$ and below the sphere $x^2 + y^2 + z^2 = 4z$.

(c) [2 pts.] Evaluate the line integral $\int_C \left(x\sqrt{y}\,dx + y\sqrt{x}\,dy\right)$ where C consists of the arc of the circle $x^2 + y^2 = 1$ from $(1,\ 0)$ to $(0,\ 1)$ and the line segment from $(0,\ 1)$ to $(0,\ 2)$.